SAMPLE QUESTION PAPER

Issued by CBSE for 2020 Examinations
Class XII - Mathematics

Time Allowed : 3 Hours **Max. Marks : 80**

General Instructions :

(a) All questions are compulsory.

(b) This question paper consists of **36 questions** divided into **four sections A, B, C and D**.

(c) Section A comprises of **20 questions of one mark** each (from Q01 – 20).
Section B comprises of **06 questions of two marks** each (from Q21 – 26).
Section C comprises of **06 questions of four marks** each (from Q27 – 32).
Section D comprises of **04 questions of six marks** each (from Q33 – 36).

(d) There is no overall choice. However, **internal choice** has been provided in **03 Questions of Section A, 02 Questions of Section B, 02 Questions of Section C** and **02 Questions of Section D, each**. You have to attempt only one of the alternatives in all such questions.

SECTION A

(Question numbers 01 to 20 carry 1 mark each.)

Followings are **multiple choice questions**. Select the correct options (from **Q01 – Q10**) :

Q01. If A is any square matrix of order 3×3 such that $|A|=3$, then the value of $|\text{adj. } A|$ is

(a) 3 (b) $\frac{1}{3}$

(c) 9 (d) 27

Q02. Suppose P and Q are two different matrices of order $3\times n$ and $n\times p$ respectively, then the order of the matrix $P\times Q$ is

(a) $3\times p$ (b) $p\times3$

(c) $n\times n$ (d) 3×3

Q03. If $(2\hat{i}+6\hat{j}+27\hat{k})\times(\hat{i}+p\hat{j}+q\hat{k})=\vec{0}$, then the value of p and q are

(a) $p=6,\ q=27$ (b) $p=3,\ q=\frac{27}{2}$

(c) $p=6,\ q=\frac{27}{2}$ (d) $p=3,\ q=27$

Q04. If A and B are two events such that $P(A)=0.2,\ P(B)=0.4$ and $P(A\cup B)=0.5$, then value of $P(A|B)$ is

(a) 0.1 (b) 0.25

(c) 0.5 (d) 0.08

Q05. The point which doesn't lie in the half plane $2x+3y-12\le0$ is

(a) (1, 2) (b) (2, 1)

(c) (2, 3) (d) (–3, 2)

Q06. If $\sin^{-1}x+\sin^{-1}y=\frac{2\pi}{3}$, then the value of $\cos^{-1}x+\cos^{-1}y$ is

(a) $\frac{2\pi}{3}$ (b) $\frac{\pi}{3}$

(c) $\frac{\pi}{2}$ (d) π

Q07. An urn contains 6 balls of which two are red and four are black. Two balls are drawn at random. Probability that they are of the different colours is

(a) $\frac{2}{5}$ (b) $\frac{1}{15}$

(c) $\frac{8}{15}$ (d) $\frac{4}{15}$

Q08. $\int \frac{dx}{\sqrt{9-25x^2}}$ equals

(a) $\sin^{-1}\left(\frac{5x}{3}\right)+C$ (b) $\frac{1}{5}\sin^{-1}\left(\frac{5x}{3}\right)+C$

(c) $\frac{1}{6}\log\left|\frac{3+5x}{3-5x}\right|+C$ (d) $\frac{1}{30}\log\left|\frac{3+5x}{3-5x}\right|+C$

Q09. What is the distance (in units) between the two planes $3x+5y+7z=3$ and $9x+15y+21z=9$?

(a) 0 (b) 3

(c) $\frac{6}{\sqrt{83}}$ (d) 6

Q10. The equation of the line in vector form passing through the point (–1, 3, 5) and parallel to line $\frac{x-3}{2}=\frac{y-4}{3}$, $z=2$, is

(a) $\vec{r}=(-\hat{i}+3\hat{j}+5\hat{k})+\lambda(2\hat{i}+3\hat{j}+\hat{k})$

(b) $\vec{r}=(-\hat{i}+3\hat{j}+5\hat{k})+\lambda(2\hat{i}+3\hat{j})$

(c) $\vec{r}=(2\hat{i}+3\hat{j}-2\hat{k})+\lambda(-\hat{i}+3\hat{j}+5\hat{k})$

(d) $\vec{r}=(2\hat{i}+3\hat{j})+\lambda(-\hat{i}+3\hat{j}+5\hat{k})$

Fill in the blanks in the following (from **Q11 – Q15**) :

Q11. If f be the greatest integer function defined as $f(x)=[x]$ and g be the modulus function defined as $g(x)=|x|$, then the value of $gof\left(-\frac{5}{4}\right)$ is ______.

Q12. If the function $f(x)=\begin{cases}\frac{x^2-1}{x-1}, & \text{when } x\neq 1\\ k, & \text{when } x=1\end{cases}$ is given to be continuous at $x=1$, then the value of k is _________.

Q13. If $\begin{bmatrix}1 & 2\\ 2 & 1\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}5\\ 4\end{bmatrix}$, then the value of y is _________.

Q14. If tangent to the curve $y^2+3x-7=0$ at the point (h, k) is parallel to the line $x-y=4$, then the value of k is____________.

OR

For the curve $y=5x-2x^3$, if x increases at the rate of 2 units/sec, then at $x=3$, the slope of the curve is changing at _________.

Q15. The magnitude of projection of $(2\hat{i}-\hat{j}+\hat{k})$ on $(\hat{i}-2\hat{j}+2\hat{k})$ is __________.

OR

Vector of magnitude 5 units and in the direction opposite to $2\hat{i}+3\hat{j}-6\hat{k}$ is__________.

Following questions are of **one word** or **short answer type** (from **Q16 – Q20**) :

Q16. Check whether $(l+m+n)$ is a factor of the determinant $\begin{vmatrix} l+m & m+n & n+l \\ n & l & m \\ 2 & 2 & 2 \end{vmatrix}$ or not. Give reason.

Q17. Evaluate $\int_{-2}^{2}(x^3+1)dx$.

Q18. Find $\int \frac{3+3\cos x}{x+\sin x}dx$.

OR

Find $\int(\cos^2 2x - \sin^2 2x)\,dx$.

Q19. Find $\int x\,e^{(1+x^2)}\,dx$.

Q20. Write the general solution of differential equation $\frac{dy}{dx} = e^{x+y}$.

SECTION B

(Question numbers 21 to 26 carry 2 marks each.)

Q21. Express $\sin^{-1}\left(\frac{\sin x + \cos x}{\sqrt{2}}\right)$, where $-\frac{\pi}{4} < x < \frac{\pi}{4}$, in the simplest form.

OR

Let R be the relation in the set Z of integers given by $R = \{(a,b) : 2 \text{ divides } a-b\}$. Show that the relation R is transitive. Write the equivalence class [0].

Q22. If $y = ae^{2x} + be^{-x}$, then show that $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 0$.

Q23. A particle moves along the curve $x^2 = 2y$. At what point, ordinate increases at the same rate as abscissa increases?

Q24. For three non-zero vectors $\vec{a}, \vec{b}$ and $\vec{c}$, prove that $[\vec{a}-\vec{b}\ \ \vec{b}-\vec{c}\ \ \vec{c}-\vec{a}] = 0$.

OR

If $\vec{a}+\vec{b}+\vec{c}=\vec{0}$ and $|\vec{a}|=3, |\vec{b}|=5, |\vec{c}|=7$ then, find the value of $\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}$.

Q25. Find the acute angle between the lines $\frac{x-4}{3} = \frac{y+3}{4} = \frac{z+1}{5}$ and $\frac{x-1}{4} = \frac{y+1}{-3} = \frac{z+10}{5}$.

Q26. A speaks truth in 80% cases and B speaks truth in 90% cases. In what percentage of cases are they likely to agree with each other in stating the same fact?

SECTION C

(Question numbers 27 to 32 carry 4 marks each.)

Q27. Let $f : A \to B$ be a function defined as $f(x) = \frac{2x+3}{x-3}$ where $A = R - \{3\}$ and $B = R - \{2\}$. Is the function f one-one and onto? Is f invertible? If yes, then find its inverse.

Q28. If $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$, then prove that $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$.

OR

If $x = a(\cos 2\theta + 2\theta\sin 2\theta)$ and $y = a(\sin 2\theta - 2\theta\cos 2\theta)$, find $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{8}$.

Q29. Solve the differential equation $xdy - ydx = \sqrt{x^2+y^2}\,dx$.

Q30. Evaluate $\int_1^3 |x^2 - 2x|\,dx$.

Q31. Two numbers are selected at random (without replacement) from first 7 natural numbers. If X denotes the smallest of the two numbers obtained, find the probability distribution of X. Also find mean of the distribution.

OR

There are three coins, one is a two headed coin (having head on both the faces), another is a biased coin that comes up heads 75% of the time and the third is an unbiased coin. One of the three coins is chosen at random and tossed. If it shows head, what is probability that it was the two headed coin?

Q32. Two tailors A and B earn ₹150 and ₹200 per day respectively. A can stitch 6 shirts and 4 pants per day, while B can stitch 10 shirts and 4 pants per day. Form an L.P.P. to minimize the labour cost to produce (stitch) at least 60 shirts and 32 pants and solve it graphically.

SECTION D

(Question numbers 33 to 36 carry 6 marks each.)

Q33. Using the properties of determinants, prove that

$$\begin{vmatrix} (y+z)^2 & x^2 & x^2 \\ y^2 & (x+z)^2 & y^2 \\ z^2 & z^2 & (x+y)^2 \end{vmatrix} = 2xyz(x+y+z)^3$$

OR

If $A = \begin{bmatrix} 2 & 3 & 4 \\ 1 & -1 & 0 \\ 0 & 1 & 2 \end{bmatrix}$, find A^{-1}.

Hence solve the following system of linear equations :

$x - y = 3,$

$2x + 3y + 4z = 17$ and,

$y + 2z = 7.$

Q34. Using integration, find the area of the region $\{(x, y) : x^2 + y^2 \le 1,\ x + y \ge 1,\ x \ge 0,\ y \ge 0\}$.

Q35. A given quantity of metal is to be cast into a solid half circular cylinder with a rectangular base and semi-circular ends. Show that in order that total surface area is minimum, the ratio of length of the cylinder to the diameter of its semi-circular ends is $\pi : (\pi + 2)$.

OR

Show that the triangle of maximum area that can be inscribed in a given circle is an equilateral triangle.

Q36. Find the equation of a plane passing through the points A(2, 1, 2) and B(4, –2, 1) and perpendicular to plane $\vec{r}.(\hat{i} - 2\hat{k}) = 5$. Also find the coordinates of the point, where the line passing through the points (3, 4, 1) and (5, 1, 6) crosses the plane thus obtained.

SOLUTIONS OF SAMPLE PAPER 2020

SECTION A

Q01. (c) $|adj.A| = |A|^{3-1} = 3^2 = 9$.

Q02. (a) As order of P and Q are respectively $3\times n$ and $n\times p$ so, clearly order of PQ is $3\times p$.

Q03. (b) As $(2\hat{i}+6\hat{j}+27\hat{k})\times(\hat{i}+p\hat{j}+q\hat{k})=\vec{0}$ so, $(2\hat{i}+6\hat{j}+27\hat{k}) \parallel (\hat{i}+p\hat{j}+q\hat{k})$

Therefore, $\frac{2}{1}=\frac{6}{p}=\frac{27}{q}$ (∵ d.r.'s of parallel vectors are proportional.

Consider $\frac{2}{1}=\frac{6}{p}$ and $\frac{2}{1}=\frac{27}{q}$

$\therefore p=3,\ q=\frac{27}{2}$.

Q04. (b) $P(A|B)=\frac{P(A\cap B)}{P(B)}=\frac{P(A)+P(B)-P(A\cup B)}{P(B)}$

$\Rightarrow P(A|B)=\frac{0.2+0.4-0.5}{0.4}=\frac{1}{4}=0.25$.

Q05. (c) Note that for only (2, 3), we have $2\times2+3\times3-12\le0$ i.e., $1\le0$, which is false.

Q06. (b) As $\sin^{-1}x+\sin^{-1}y=\frac{2\pi}{3}$

$\Rightarrow \frac{\pi}{2}-\cos^{-1}x+\frac{\pi}{2}-\cos^{-1}y=\frac{2\pi}{3}$

$\Rightarrow \cos^{-1}x+\cos^{-1}y=\pi-\frac{2\pi}{3}=\frac{\pi}{3}$.

Q07. (c) Required probability $=\frac{{}^2C_1\times{}^4C_1}{{}^6C_2}=\frac{2\times4\times2}{6\times5}=\frac{8}{15}$.

Q08. (b) $\int\frac{dx}{\sqrt{9-25x^2}}=\int\frac{dx}{\sqrt{3^2-(5x)^2}}=\frac{1}{5}\sin^{-1}\frac{5x}{3}+C$.

Q09. (a) Note that the d.r.'s of normal to both the planes are proportional i.e., $\frac{3}{9}=\frac{5}{15}=\frac{7}{21}$.

That means, the given planes are parallel.
Re-writing the planes, we have $3x+5y+7z-3=0$ and $3x+5y+7z-3=0$
Clearly, distance between these equations of planes will be **zero** as they represent same plane.

Also $p=\frac{|D_1-D_2|}{\sqrt{A^2+B^2+C^2}}=\frac{|-3-(-3)|}{\sqrt{3^2+5^2+7^2}}=0$ units.

Q10. (b) Re-writing the given line : $\frac{x-3}{2}=\frac{y-4}{3}=\frac{z-2}{0}$ [∵ $z=z_1+c\lambda$, parametric eq.

The d.r.'s of this line : 2, 3, 0.
So the required line through (–1, 3, 5) is : $\vec{r}=(-\hat{i}+3\hat{j}+5\hat{k})+\lambda(2\hat{i}+3\hat{j})$.
Note that the d.r.'s of parallel lines are proportional.

Q11. $gof\left(-\frac{5}{4}\right)=g\left(\left[-\frac{5}{4}\right]\right)=g(-2)=|-2|=2$.

Q12. As $f(x)$ is continuous at $x=1$ so, $\lim_{x\to 1} f(x) = f(1)$ i.e., $\lim_{x\to 1}\frac{x^2-1}{x-1} = k$

$\Rightarrow \lim_{x\to 1}\frac{(x-1)(x+1)}{x-1} = k$

$\Rightarrow \lim_{x\to 1}(x+1) = k \qquad \therefore k = 2$.

Q13. $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 4 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} x+2y \\ 2x+y \end{bmatrix} = \begin{bmatrix} 5 \\ 4 \end{bmatrix}$

By equality of matrices, we get : $x+2y=5,\ 2x+y=4$

On solving these equations, we get : $y = 2$.

Q14. As $\frac{dy}{dx} = -\frac{3}{2y}$

$\therefore \left.\frac{dy}{dx}\right]_{\text{at }(h,k)} = -\frac{3}{2k} = 1$

Therefore, $k = -\frac{3}{2}$

Note that the slope of $x-y=4$ is 1 and also that, slope of parallel lines will be same always.

OR

Slope of curve is, $m = \frac{dy}{dx} = 5-6x^2$

$\Rightarrow \frac{dm}{dt} = 0 - 12x \times \frac{dx}{dt}$

$\therefore \left.\frac{dm}{dt}\right]_{\text{at } x=3} = -12\times 3\times 2 = -72$ units/ sec

Therefore, the slope of the curve is decreasing at the rate of 72 units/sec.

Q15. Projection of $(2\hat{i}-\hat{j}+\hat{k})$ on $(\hat{i}-2\hat{j}+2\hat{k})$ is $(2\hat{i}-\hat{j}+\hat{k}).\left(\frac{\hat{i}-2\hat{j}+2\hat{k}}{\sqrt{1+4+4}}\right) = \frac{2+2+2}{3} = 2$.

OR

Let $\vec{r} = 2\hat{i}+3\hat{j}-6\hat{k}$

$\therefore$ Required vector $= 5\left(\frac{-\vec{r}}{|\vec{r}|}\right) = 5\left(\frac{-2\hat{i}-3\hat{j}+6\hat{k}}{\sqrt{4+9+36}}\right) = \frac{5}{7}(-2\hat{i}-3\hat{j}+6\hat{k})$.

Q16. Let $\Delta = \begin{vmatrix} l+m & m+n & n+l \\ n & l & m \\ 2 & 2 & 2 \end{vmatrix}$

Applying $R_1 \to R_1 + R_2$, we get $\Delta = \begin{vmatrix} l+m+n & l+m+n & l+m+n \\ n & l & m \\ 2 & 2 & 2 \end{vmatrix}$

Taking $(l+m+n)$ and 2 common from R_1 and R_3 respectively, we get

$$\Delta = 2(l+m+n)\begin{vmatrix} 1 & 1 & 1 \\ n & l & m \\ 1 & 1 & 1 \end{vmatrix}.$$

Clearly, $(l+m+n)$ is a factor of the determinant, Δ.

Q17. Let $I = \int_{-2}^{2} (x^3+1)dx$

$\Rightarrow I = \int_{-2}^{2} x^3 dx + \int_{-2}^{2} 1\,dx$

Consider $f(x) = x^3$ $\quad \therefore f(-x) = -x^3 = -f(x)$ i.e., f (x) is an odd function.

And, $g(x) = 1$ $\quad \therefore g(-x) = 1 = g(x)$ i.e., g (x) is an even function.

So, $I = 0 + 2\int_{0}^{2} 1\,dx = 2\left[x\right]_0^2$

$\therefore I = 2\left[2-0\right] = 4$.

Q18. We have $\int \frac{3+3\cos x}{x+\sin x}dx = 3\int \frac{1+\cos x}{x+\sin x}dx = 3\log\left|x+\sin x\right| + C$ $\quad \left[\because \int \frac{f'(x)}{f(x)}dx = \log\left|f(x)\right| + C\right.$

OR

We have $\int (\cos^2 2x - \sin^2 2x)\,dx = \int \cos 4x\,dx = \frac{1}{4}\sin 4x + C$.

Q19. Put $1+x^2 = y$ $\quad \Rightarrow x dx = \frac{dy}{2}$

$\therefore \int x\, e^{(1+x^2)}\,dx = \frac{1}{2}\int e^y\,dy = \frac{1}{2}\times e^y + C = \frac{1}{2}\times e^{(1+x^2)} + C$

Q20. We have $\frac{dy}{dx} = e^{x+y}$

$\Rightarrow \frac{dy}{dx} = e^x \times e^y$

$\Rightarrow \int e^{-y}dy = \int e^x dx$

$\Rightarrow -e^{-y} = e^x + k$

Therefore, $e^x + e^{-y} = C$, where $C = -k$, is the required general solution.

SECTION B

Q21. Let $y = \sin^{-1}\left(\frac{\sin x + \cos x}{\sqrt{2}}\right)$

$\Rightarrow y = \sin^{-1}\left(\frac{1}{\sqrt{2}}\sin x + \frac{1}{\sqrt{2}}\cos x\right)$

$\Rightarrow y = \sin^{-1}\left(\sin x\cos\frac{\pi}{4} + \cos x \sin\frac{\pi}{4}\right)$

$\Rightarrow y = \sin^{-1}\sin\left(x+\frac{\pi}{4}\right)$

$\therefore y = x + \frac{\pi}{4}$

As $-\frac{\pi}{4} < x < \frac{\pi}{4} \Rightarrow 0 < \frac{\pi}{4} + x < \frac{\pi}{2}$ i.e., $\left(x+\frac{\pi}{4}\right) \in \left(0, \frac{\pi}{2}\right)$, which corresponds to principal branch of $\sin^{-1} x$.

Also $\sin^{-1} \sin x = x,\ x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

OR

Let a, b, $c \in Z$. Let $(a,b) \in R$ and $(b,c) \in R$.
That is, 2 divides (a – b) and 2 divides (b – c).
So, 2 must divide (a – b) + (b – c) $= a - c$.
That implies, 2 divides (a – c).
That is, $(a,c) \in R$.
Therefore, R is transitive.
For [0], let $(x,0) \in R\ \forall\ x \in Z$.
That is, 2 divides $x - 0 \Rightarrow x = 0, \pm 2, \pm 4, \pm 6, \ldots$.
Hence, $[0] = \{0, \pm 2, \pm 4, \pm 6, \ldots\}$.

Q22. We've $y = ae^{2x} + be^{-x}$ …(i)

$\Rightarrow \frac{dy}{dx} = 2ae^{2x} - be^{-x}$ …(ii)

$\Rightarrow \frac{d^2y}{dx^2} = 4ae^{2x} + be^{-x}$ …(ii)

LHS : $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = \left(4ae^{2x} + be^{-x}\right) - \left(2ae^{2x} - be^{-x}\right) - 2\left(ae^{2x} + be^{-x}\right)$ [By (i), (ii) and (iii)

$\Rightarrow \frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = \left(4ae^{2x} + 2be^{-x}\right) - \left(4ae^{2x} + 2be^{-x}\right) = 0 = \text{RHS}$.

Q23. We have $x^2 = 2y$ ..(i)

$\Rightarrow 2x \times \frac{dx}{dt} = 2 \times \frac{dy}{dt}$

$\therefore 2x \times \frac{dx}{dt} = 2 \times \frac{dx}{dt}$ $\left[\because \text{Given that } \frac{dy}{dt} = \frac{dx}{dt}\right.$

$\Rightarrow x = 1$

By (i), $2y = 1^2$ $\Rightarrow y = \frac{1}{2}$.

So, the required point is $\left(1, \frac{1}{2}\right)$.

Q24. LHS : $[\vec{a}-\vec{b}\ \ \vec{b}-\vec{c}\ \ \vec{c}-\vec{a}] = \left[(\vec{a}-\vec{b}) \times (\vec{b}-\vec{c})\right].(\vec{c}-\vec{a})$

$\Rightarrow \quad = \left[\vec{a}\times\vec{b} - \vec{a}\times\vec{c} - \vec{b}\times\vec{b} + \vec{b}\times\vec{c}\right].(\vec{c}-\vec{a})$

$\Rightarrow \quad = (\vec{a}\times\vec{b}).\vec{c} - (\vec{a}\times\vec{b}).\vec{a} - (\vec{a}\times\vec{c}).\vec{c} + (\vec{a}\times\vec{c}).\vec{a} + (\vec{b}\times\vec{c}).\vec{c} - (\vec{b}\times\vec{c}).\vec{a}$ $[\because \vec{b}\times\vec{b} = \vec{0}$

$\Rightarrow \quad = (\vec{a}\times\vec{b}).\vec{c} - (\vec{b}\times\vec{c}).\vec{a}$

$\Rightarrow \quad = [\vec{a}\ \ \vec{b}\ \ \vec{c}] - [\vec{b}\ \ \vec{c}\ \ \vec{a}] = [\vec{a}\ \ \vec{b}\ \ \vec{c}] - [\vec{a}\ \ \vec{b}\ \ \vec{c}] = 0 = \text{RHS}$.

OR

Here $\vec{a}+\vec{b}+\vec{c}=\vec{0}$

$\Rightarrow (\vec{a}+\vec{b}+\vec{c}).(\vec{a}+\vec{b}+\vec{c})=\vec{0}.\vec{0}$

$\Rightarrow \vec{a}.\vec{a}+\vec{a}.\vec{b}+\vec{a}.\vec{c}+\vec{b}.\vec{a}+\vec{b}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}+\vec{c}.\vec{b}+\vec{c}.\vec{c}=0$

$\Rightarrow |\vec{a}|^2+|\vec{b}|^2+|\vec{c}|^2+2\vec{a}.\vec{b}+2\vec{b}.\vec{c}+2\vec{c}.\vec{a}=0$

$\Rightarrow 3^2+5^2+7^2+2(\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a})=0$

$\therefore (\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a})=-\frac{83}{2}$.

Q25. The d.r.'s of the given lines $\frac{x-4}{3}=\frac{y+3}{4}=\frac{z+1}{5}$ and $\frac{x-1}{4}=\frac{y+1}{-3}=\frac{z+10}{5}$ are $3,4,5$ and $4,-3,5$ respectively.

Using $\cos\theta=\frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}$, where θ is the required angle between given lines

$\Rightarrow \cos\theta=\frac{3\times4+4\times(-3)+5\times5}{\sqrt{3^2+4^2+5^2}\sqrt{4^2+(-3)^2+5^2}}$

$\Rightarrow \cos\theta=\frac{25}{5\sqrt{2}\times5\sqrt{2}}=\frac{1}{2}$

$\therefore \theta=\frac{\pi}{3}$.

Q26. Let A and B denote the event that A speaks truth and B speaks the truth, respectively.

We have $P(A)=80\%,\ P(B)=90\%$ $\therefore P(\overline{A})=100\%-80\%=20\%,\ P(\overline{B})=10\%$.

Therefore, P(A and B agree) $=$ P(both speak truth or both lie)

$\Rightarrow \qquad = P(AB \text{ or } \overline{A}\overline{B})=P(A)P(B)+P(\overline{A})P(\overline{B})$

$\therefore P(\text{A and B agree})=\frac{80}{100}\times\frac{90}{100}+\frac{20}{100}\times\frac{10}{100}=\frac{74}{100}$ i.e., 74%.

Therefore, in 74% of the cases A and B are likely to agree with each other in stating the same fact.

SECTION C

Q27. Let $y=f(x)=\frac{2x+3}{x-3}$.

Let $\alpha,\beta\in A=R-\{3\}$.

Let $f(\alpha)=f(\beta)$ i.e., $\frac{2\alpha+3}{\alpha-3}=\frac{2\beta+3}{\beta-3}$

$\Rightarrow 2\alpha\beta-6\alpha+3\beta-9=2\alpha\beta-6\beta+3\alpha-9$

$\Rightarrow -9\alpha=-9\beta$

$\Rightarrow \alpha=\beta$

$\therefore$ f(x) is one-one.

Also, $y=\frac{2x+3}{x-3}$

$\Rightarrow xy-3y=2x+3$

$\Rightarrow xy-2x=3y+3$

$\Rightarrow x=\frac{3y+3}{y-2}$ …(i)

Note that (i) is defined iff $y\in R-2=B$.

Also $f(x)=f\left(\frac{3y+3}{y-2}\right)=\frac{2\left(\frac{3y+3}{y-2}\right)+3}{\left(\frac{3y+3}{y-2}\right)-3}=y$

That is, $x\in A\ \forall\ y\in B$ implying codomain = range.

Hence f is onto.

As f (x) is one-one and onto both so, it's bijective and hence f (x) is invertible.

Also, inverse of f (x) is given by $f^{-1}(y)=\frac{3y+3}{y-2}$ i.e., $f^{-1}(x)=\frac{3x+3}{x-2}$.

Q28. Put $x=\sin\alpha, y=\sin\beta \Rightarrow \alpha=\sin^{-1}x, \beta=\sin^{-1}y$ …(i)

$\therefore \sqrt{1-x^2}+\sqrt{1-y^2}=a(x-y)$

$\Rightarrow \sqrt{1-\sin^2\alpha}+\sqrt{1-\sin^2\beta}=a(\sin\alpha-\sin\beta)$

$\Rightarrow \cos\alpha+\cos\beta=2a\left(\cos\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}\right)$

$\Rightarrow 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}=2a\cos\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$

$\Rightarrow \cot\frac{\alpha-\beta}{2}=a$

$\Rightarrow \frac{\alpha-\beta}{2}=\cot^{-1}a$

$\Rightarrow \alpha-\beta=2\cot^{-1}a$

$\Rightarrow \sin^{-1}x-\sin^{-1}y=2\cot^{-1}a$ [By (i)

$\therefore \frac{1}{\sqrt{1-x^2}}-\frac{1}{\sqrt{1-y^2}}\times\frac{dy}{dx}=0$

Therefore, $\frac{dy}{dx}=\sqrt{\frac{1-y^2}{1-x^2}}$.

OR

Here $x=a(\cos 2\theta+2\theta\sin 2\theta)$ and $y=a(\sin 2\theta-2\theta\cos 2\theta)$

$\therefore \frac{dx}{d\theta}=a(-2\sin 2\theta+4\theta\cos 2\theta+2\sin 2\theta)=4a\theta\cos 2\theta$

and $\frac{dy}{d\theta}=a(2\cos 2\theta+4\theta\sin 2\theta-2\cos 2\theta)=4a\theta\sin 2\theta$

So, $\frac{dy}{dx}=\frac{dy}{d\theta}\times\frac{d\theta}{dx}=\frac{4a\theta\sin 2\theta}{4a\theta\cos 2\theta}=\tan 2\theta$

$\Rightarrow \frac{d}{dx}\left(\frac{dy}{dx}\right)=\frac{d^2y}{dx^2}=2\sec^2 2\theta\times\frac{d\theta}{dx}$

$\Rightarrow \frac{d^2y}{dx^2}=2\sec^2 2\theta\times\frac{1}{4a\theta\cos 2\theta}$

$\Rightarrow \frac{d^2y}{dx^2}=\frac{\sec^3 2\theta}{2a\theta}$

$\Rightarrow \left.\frac{d^2y}{dx^2}\right]_{\text{at }\theta=\frac{\pi}{8}}=\frac{\sec^3\frac{\pi}{4}}{a\left(\frac{\pi}{4}\right)}=\frac{4}{a\pi}\times 2\sqrt{2}$

$$\therefore \left.\frac{d^2y}{dx^2}\right]_{\text{at } \theta=\frac{\pi}{8}} = \frac{8\sqrt{2}}{a\pi}.$$

Q29. $x\,dy - y\,dx = \sqrt{x^2+y^2}\,dx$

$$\Rightarrow \frac{dy}{dx} - \frac{y}{x} = \frac{\sqrt{x^2+y^2}}{x}$$

$$\Rightarrow \frac{dy}{dx} = \frac{\sqrt{x^2+y^2}+y}{x} \qquad \ldots(i)$$

Put $y = vx$

$$\Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$$

By (i), we get : $v + x\dfrac{dv}{dx} = \dfrac{\sqrt{x^2+v^2x^2}+vx}{x}$

$$\Rightarrow v + x\frac{dv}{dx} = \sqrt{1+v^2}+v$$

$$\Rightarrow \int \frac{dv}{\sqrt{1+v^2}} = \int \frac{dx}{x}$$

$$\Rightarrow \log\left(v+\sqrt{1+v^2}\right) = \log x + \log C$$

$$\Rightarrow \log\left(\frac{y}{x}+\sqrt{1+\frac{y^2}{x^2}}\right) = \log(Cx)$$

$$\Rightarrow \left(\frac{y+\sqrt{x^2+y^2}}{x}\right) = Cx$$

$\therefore y+\sqrt{x^2+y^2} = Cx^2$ is the required solution.

Q30. Consider $f(x) = |x^2-2x|$

$$\Rightarrow f(x) = |x(x-2)| = \begin{cases} -(x^2-2x), & \text{if } 1 \le x < 2 \\ x^2-2x, & \text{if } 2 \le x \le 3 \end{cases}$$

Now let $I = \int_1^3 f(x)\,dx$

$$\Rightarrow I = \int_1^2 f(x)\,dx + \int_2^3 f(x)\,dx$$

$$\Rightarrow I = -\int_1^2 (x^2-2x)\,dx + \int_2^3 (x^2-2x)\,dx$$

$$\Rightarrow I = -\left[\frac{x^3}{3}-x^2\right]_1^2 + \left[\frac{x^3}{3}-x^2\right]_2^3$$

$$\Rightarrow I = -\left[\left(\frac{8}{3}-4\right)-\left(\frac{1}{3}-1\right)\right] + \left[(9-9)-\left(\frac{8}{3}-4\right)\right]$$

$$\Rightarrow I = -\left[-\frac{4}{3}+\frac{2}{3}\right] + \left[\frac{4}{3}\right]$$

$$\Rightarrow I = -\left[-\frac{2}{3}\right] + \left[\frac{4}{3}\right]$$

$\Rightarrow I = \frac{6}{3}$

$\Rightarrow I = 2$.

Q31. Here X denotes the smallest of the two numbers obtained from first 7 natural numbers.

So, X can take values 1, 2, 3, 4, 5, 6.

Total no. of possible ways in which 2 nos. can be selected from 7 natural nos. is $^7C_2 = 21$.

Table for probability distribution is given as below :

X	P(X)
1	$\frac{^6C_1}{21} = \frac{6}{21}$
2	$\frac{^5C_1}{21} = \frac{5}{21}$
3	$\frac{^4C_1}{21} = \frac{4}{21}$
4	$\frac{^3C_1}{21} = \frac{3}{21}$
5	$\frac{^2C_1}{21} = \frac{2}{21}$
6	$\frac{^1C_1}{21} = \frac{1}{21}$

Now mean, $E(X) = \sum X\ P(X) = 1\times\frac{6}{21} + 2\times\frac{5}{21} + 3\times\frac{4}{21} + 4\times\frac{3}{21} + 5\times\frac{2}{21} + 6\times\frac{1}{21} = \frac{56}{21} = \frac{8}{3}$.

OR

Let E : the coin shows head.

Also let E_1, E_2, E_3 : the coin is a two headed, a biased coin and unbiased coin, respectively.

Here $P(E_1) = P(E_2) = P(E_3) = \frac{1}{3}$, $P(E|E_1) = 1$, $P(E|E_2) = 75\% = \frac{3}{4}$, $P(E|E_3) = \frac{1}{2}$.

By Bayes' Theorem, $P(E_1 | E) = \frac{P(E|E_1)P(E_1)}{P(E|E_1)P(E_1) + P(E|E_2)P(E_2) + P(E|E_3)P(E_3)}$

$$\Rightarrow P(E_1 | E) = \frac{1\times\frac{1}{3}}{1\times\frac{1}{3} + \frac{3}{4}\times\frac{1}{3} + \frac{1}{2}\times\frac{1}{3}}$$

$\therefore P(E_1 | E) = \frac{4}{9}$.

Q32. Let the number of days for which the tailors A and B work be x and y, respectively.

To minimize : $Z = ₹(150x + 200y)$

Subject to constraints :

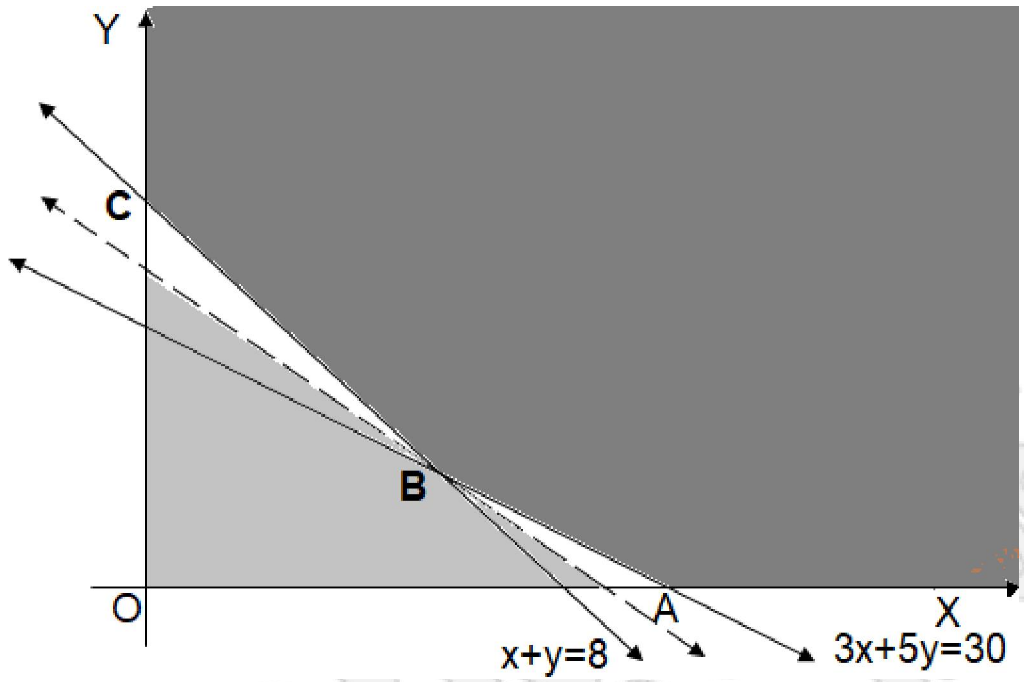
$6x+10y\geq 60,$

$4x+4y\geq 32,$

$x\geq 0,$

$y\geq 0$

That is, $3x+5y\geq 30,\ x+y\geq 8;\ x,y\geq 0$



Corner Points	Value of Z (in ₹)
A(10, 0)	1500
B(5, 3)	$1350 \leftarrow$ Minimum value
C(0, 8)	1600

Since the feasible region is unbounded so, $Z=1350$ may or may not be the minimum value.
To check, let $150x+200y<1350$ i.e., $3x+4y<27$.
As there's no common point between $3x+4y<27$ and the feasible region so, $Z=1350$ is the minimum value.

SECTION D

Q33. LHS : Let $\Delta=\begin{vmatrix}(y+z)^2 & x^2 & x^2\\ y^2 & (x+z)^2 & y^2\\ z^2 & z^2 & (x+y)^2\end{vmatrix}$

By $C_2\to C_2-C_1$, $C_3\to C_3-C_1$

$$\Rightarrow \Delta=\begin{vmatrix}(y+z)^2 & x^2-(y+z)^2 & x^2-(y+z)^2\\ y^2 & (x+z)^2-y^2 & 0\\ z^2 & 0 & (x+y)^2-z^2\end{vmatrix}$$

Take $x+y+z$ common from C_2 and C_3 both

$$\Rightarrow \Delta = (x+y+z)^2 \begin{vmatrix} (y+z)^2 & x-y-z & x-y-z \\ y^2 & x+z-y & 0 \\ z^2 & 0 & x+y-z \end{vmatrix}$$

By $R_1 \to R_1 - (R_2 + R_3)$

$$\Rightarrow \Delta = (x+y+z)^2 \begin{vmatrix} 2yz & -2z & -2y \\ y^2 & x-y+z & 0 \\ z^2 & 0 & x+y-z \end{vmatrix}$$

By $C_2 \to C_2 + \frac{1}{y}C_1$ and $C_3 \to C_3 + \frac{1}{z}C_1$

$$\Rightarrow \Delta = (x+y+z)^2 \begin{vmatrix} 2yz & 0 & 0 \\ y^2 & x+z & \frac{y^2}{z} \\ z^2 & \frac{z^2}{y} & x+y \end{vmatrix}$$

Expanding along R_1

$$\Rightarrow \Delta = (x+y+z)^2 \left\{2yz\left(x^2+xy+zx+zy-zy\right)-0+0\right\}$$

$$\Rightarrow \Delta = (x+y+z)^2\, 2yzx(x+y+z)$$

$$\therefore \Delta = 2xyz(x+y+z)^3 = \text{RHS}.$$

OR

We have $A = \begin{bmatrix} 2 & 3 & 4 \\ 1 & -1 & 0 \\ 0 & 1 & 2 \end{bmatrix}$

$$\therefore |A| = \begin{vmatrix} 2 & 3 & 4 \\ 1 & -1 & 0 \\ 0 & 1 & 2 \end{vmatrix} = -4-6+4 = -6 \neq 0.$$

So, A^{-1} exists.

Let A_{ij} be the cofactor of element a_{ij} of matrix A.

$A_{11} = -2,\ A_{12} = -2,\ A_{13} = 1,$

$A_{21} = -2,\ A_{22} = 4,\ A_{23} = -2,$

$A_{31} = 4,\ A_{32} = 4,\ A_{33} = -5$

So, $\text{adj}.A = \begin{bmatrix} -2 & -2 & 4 \\ -2 & 4 & 4 \\ 1 & -2 & -5 \end{bmatrix}$ $\quad \therefore A^{-1} = \frac{\text{adj}.A}{|A|} = -\frac{1}{6}\begin{bmatrix} -2 & -2 & 4 \\ -2 & 4 & 4 \\ 1 & -2 & -5 \end{bmatrix}$

Now the system of equation can be re-written as $2x+3y+4z=17,\ x-y=3,\ y+2z=7.$

These equations can be written as $AX = B$, where $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and, $B = \begin{bmatrix} 17 \\ 3 \\ 7 \end{bmatrix}$.

So, $X = A^{-1}B$

$$\Rightarrow X = -\frac{1}{6}\begin{bmatrix} -2 & -2 & 4 \\ -2 & 4 & 4 \\ 1 & -2 & -5 \end{bmatrix}\begin{bmatrix} 17 \\ 3 \\ 7 \end{bmatrix}$$

$$\Rightarrow X = -\frac{1}{6}\begin{bmatrix} -12 \\ 6 \\ -24 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}$$

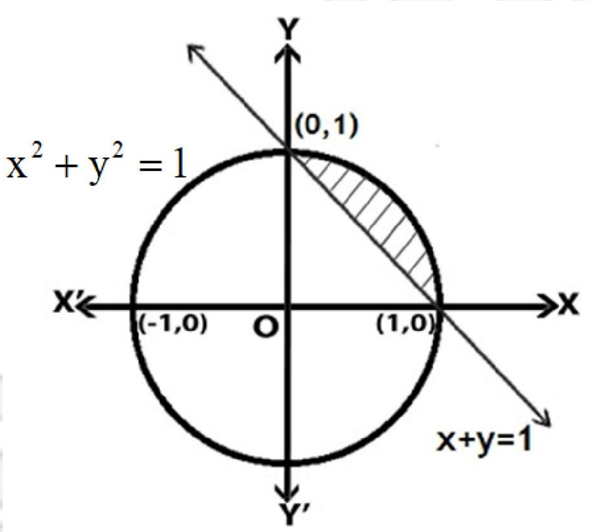
By equality of matrices, we get : $x = 2,\ y = -1,\ z = 4$.

Q34. We have $\left\{(x, y) : x^2 + y^2 \le 1,\ x + y \ge 1,\ x \ge 0,\ y \ge 0\right\}$

Consider $x^2 + y^2 = 1 ...(i),\ x + y = 1 ...(ii)$ and $x = 0,\ y = 0$.

Curve (i) is a circle whose centre is at origin and radius is of 1 unit.

Also the line (ii) cuts off the intercepts of 1 unit on both the axes in the 1st quadrant.



$$\therefore \text{Required area} = \int_0^1 y_i dx - \int_0^1 y_{ii} dx$$

$$\Rightarrow \quad = \int_0^1 \sqrt{1 - x^2} dx - \int_0^1 (1 - x) dx$$

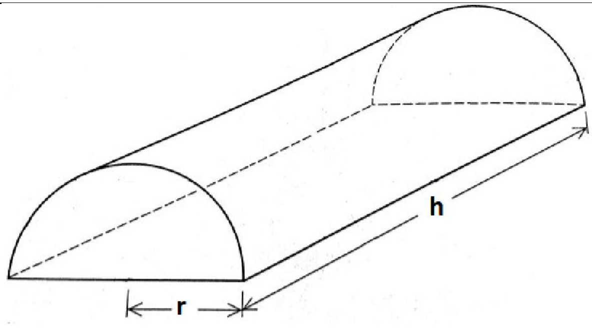
$$\Rightarrow \quad = \left[\frac{x}{2}\sqrt{1 - x^2} + \frac{1}{2}\sin^{-1} x\right]_0^1 + \frac{1}{2}\left[(1 - x)^2\right]_0^1$$

$$\Rightarrow \quad = \left[0 + \frac{1}{2}\times\frac{\pi}{2}\right] - 0 + \frac{1}{2}[0 - 1]$$

$$\Rightarrow \quad = \left(\frac{\pi}{4} - \frac{1}{2}\right) \text{Sq. units}.$$

Q35. Given volume $V = \frac{1}{2}\pi r^2 h$

$$\Rightarrow h = \frac{2V}{\pi r^2} ...(i)$$



Total surface area of the half cylinder, $A = \pi r h + \frac{1}{2}\pi r^2 + \frac{1}{2}\pi r^2 + 2rh$

$\Rightarrow A = \frac{2V}{r} + \pi r^2 + \frac{4V}{\pi r}$ $\quad$ $\left[\text{using } h = \frac{2V}{\pi r^2} \text{ from (i)}\right.$

$\therefore \frac{dA}{dr} = -\frac{2V}{r^2} + 2\pi r - \frac{4V}{\pi r^2}$ and, $\frac{d^2A}{dr^2} = \frac{4V}{r^3} + 2\pi + \frac{8V}{\pi r^3}$

For local points of maxima and/or minima, $\frac{dA}{dr} = 2\left(\pi r - \frac{V}{r^2} - \frac{2V}{\pi r^2}\right) = 0$

$\Rightarrow \pi^2 r^3 = V\pi + 2V$ $\qquad \therefore r = \left[\frac{V(\pi+2)}{\pi^2}\right]^{1/3}$

$\because \left.\frac{d^2A}{dr^2}\right]_{\text{at } r=\left[\frac{V(\pi+2)}{\pi^2}\right]^{1/3}} = \frac{4V\pi^2}{V\pi + 2V} + 2\pi + \frac{8V\pi}{V\pi + 2V} > 0$

$\therefore$ A is minimum at $r = \left[\frac{V(\pi+2)}{\pi^2}\right]^{1/3}$

Now, $\pi^2 r^3 = V\pi + 2V$

$\Rightarrow \pi^2 r^3 = [\pi + 2]\left(\frac{\pi r^2 h}{2}\right)$

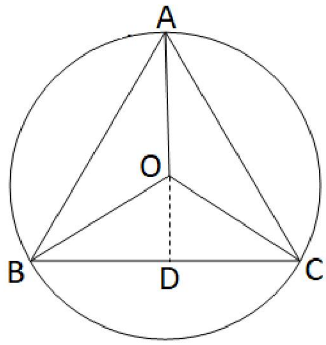
$\Rightarrow \pi r = [\pi + 2]\left(\frac{h}{2}\right)$

$\Rightarrow \frac{\pi}{\pi + 2} = \left(\frac{h}{2r}\right)$

$\therefore$ Length of cylinder : Diameter of cylinder $= \pi : (\pi + 2)$.

OR

Let r be the radius of the circle and, ABC be the triangle inscribed in it.
Let $\angle BOD = \theta \quad \therefore \angle COD = \theta$.
Also, let OD = x. So, AD = r + x .
In ΔODB, $BD = \sqrt{r^2 - x^2}$. Also, $2BD = BC = 2\sqrt{r^2 - x^2}$.



$\therefore$ ar(ABC), $A = \frac{1}{2} \times 2\sqrt{r^2 - x^2} \times (r + x)$

$\Rightarrow A^2 = f(x) = (r^2 - x^2)(r + x)^2$

$\Rightarrow f'(x) = 2(r^2 - x^2)(r + x) - 2x(r + x)^2$

$\Rightarrow f'(x) = [2r - 4x](r + x)^2$

$\Rightarrow f''(x) = 2[2r - 4x](r + x) - 4(r + x)^2$

$\Rightarrow f''(x) = -12x(r + x)$

For local points of maxima and/or minima, $f'(x) = [2r - 4x](r+x)^2 = 0 \quad \Rightarrow x = \frac{r}{2},\ x \neq -r$

$\therefore f''\left(\frac{r}{2}\right) = -6r(3r/2) < 0$

$\therefore f(x)$ and hence A is maximum at $x = \frac{r}{2}$

Now, $OD = \frac{r}{2},\ BD = \sqrt{r^2 - x^2} = \frac{r\sqrt{3}}{2}$

In $\Delta ODB,\ \frac{OD}{BD} = \cot\theta \qquad \Rightarrow \cot\theta = \frac{r/2}{r\sqrt{3}/2} = \frac{1}{\sqrt{3}} \qquad \therefore \theta = \frac{\pi}{3}$

$\Rightarrow 2\theta = \angle BOC = \frac{2\pi}{3}$

$\because \angle BAC = \frac{1}{2}\angle BOC = \frac{\pi}{3}$

$\therefore \Delta ABC$ is an equilateral triangle ($\because$ all the angles in an equilateral triangle are of measure $\frac{\pi}{3}$).

Q36. Let A, B, C be the d.r.'s of normal to the plane through the points A(2, 1, 2) and B(4, –2, 1).
$\therefore$ Eq. of plane, $\pi : A(x-2) + B(y-1) + C(z-2) = 0 ...(i)$

Also, $A(4-2) + B(-2-1) + C(1-2) = 0$ that is, $2A - 3B - C = 0 ...(ii)$

Since (i) is perpendicular to plane $\vec{r}.(\hat{i} - 2\hat{k}) = 5$ so, using $A_1A_2 + B_1B_2 + C_1C_2 = 0$ for perpendicular planes, we get : $A - 2C = 0 ...(iii)$

Solving (ii) and (iii), we get : $A = 2C,\ B = C$.

Therefore, the d.r.'s of normal to plane (i) are A, B, C i.e., $2C, C, C$ i.e., $2, 1, 1$.

By (i), required plane is : $2(x-2) + 1(y-1) + 1(z-2) = 0$ i.e., $2x + y + z = 7$ ...(iv)

Now the line passing through the points (3, 4, 1) and (5, 1, 6) is : $\frac{x-3}{2} = \frac{y-4}{-3} = \frac{z-1}{5} = \lambda$.

The coordinates of any random point on this line is $(2\lambda + 3,\ -3\lambda + 4,\ 5\lambda + 1)$.

For the required point of intersection, this point must satisfy (iv) i.e.,

$2(2\lambda + 3) + (-3\lambda + 4) + (5\lambda + 1) = 7 \qquad \Rightarrow \lambda = -\frac{2}{3}$.

$\therefore$ the required point of intersection is $\left(2\left(-\frac{2}{3}\right) + 3,\ -3\left(-\frac{2}{3}\right) + 4,\ 5\left(-\frac{2}{3}\right) + 1\right)$ i.e., $\left(\frac{5}{3},\ 6,\ -\frac{7}{3}\right)$.

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This Sample Paper has been issued by **CBSE, New Delhi** for **2019 Board Exams of XII**.
Note : We've **re-typed** the same and have **added more illustrations** in the solutions.
On other hand, if you find any error which could have gone un-noticed, please do inform us via **WhatsApp @ +919650350480** or **Email us : iMathematicia@gmail.com**
For video lectures, please visit YouTube.com/@theopgupta